ROTATORY MOTION

1. When a rigid body rotates about a fixed axis every particle of the body moves in circular path.
2. The perpendicular distance from axis of ro tation to given point is called radius vector.
3. In rotation of a body about a fixed axis angular variables are same for all particles but linear variables changes
   Relation between angular and linear variables.
   1. $v = r\omega$
   2. $a = r\alpha$
4. Kinematical Equations of rotatory Motion:
   1. $\omega = \omega_0 + \alpha t$
   2. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
   3. $\omega^2 - \omega_0^2 = 2\alpha\theta$

   When ω_0 : Initial angular velocity
   ω : Final angular velocity
   α : Uniform angular acceleration
   t : time

   Important Points:
5. **Torque:**
   The turning effect of a force about the axis of rotation is called moment of force or torque.

   Torque = Force x Perpendicular distance of line of action of force from axis of rotation.

$$\overline{\tau} = \overline{r} \times \overline{F}$$

$$|\overline{\tau}| = rF\sin\theta$$

$\overline{r}$: Position vector
6. **Couple:**
   Two forces equal in magnitude but opposite in direction acting at two different points of a body constitute coupe
7. **Moment of Couple:**
   The product of magnitude of force in couple and perpendicular distance between them.
8. **Moment of Inertia:**

Def : Inability of a body to change its state of rotation by itself is called moment of inertia (I). Moment of inertia is analogue to mass in translatory motion.

Mathematical Definition:

Moment of inertia of a rigid body about a given axis of rotation is the sum of products of the masses of various particles and square of their perpendicular distance from the axis of rotation

$I = m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2$ or $I = \sum_{i=1}^{n} m_i . r_i^2$

Units Kg.m² Dimensions: $M L^2 T^0$

9. Moment of inertia of different bodies
   a. Rod – axis Passing through its centre and perpendicular to its lenght $\frac{M\ell^2}{12}$
   b. Rectangular plate or bar – axis Passing through its center and perpenducular to plane $\frac{M\ell^2}{12} + \frac{Mb^2}{12}$
   c. Circular disc – axis Passign through its centre and perpendicular to its plane or transverse axis $\frac{MR^2}{2}$
   d. Solid cylinder – Its own axis $\frac{MR^2}{2}$
10. **Radius of gyration:**
    It is defined as the distance from the axis of rotation to a point where whole mass of the rotating body supposed to be concentrated.
    It is denoted by 'K'
    If 'K' is radius of gyration
    Then $I = mK^2$

Important Points:

a. Moment of inertia of a body depends on mass of the body and its distribution about the axis of rotation.
b. Moment of inertia changes with change in position of axis of rotation
c. Radius of gyration is not a constant quantity. Its value changes with change in location of axis of rotation.

Relation between τ *and* I . $\tau = I\alpha$.

(It is analogue to F = ma in translatory motion).

11. **Angular momentum (L):**
Moment of linear momentum of a particle in rotation about axis of rotation is known as Angular momentum
Angular momentum = linear momentum x Perpendicular distance from axis of rotation.
Angular momentum is a vector quantity
In vector from $\overline{L} = \overline{r} \times \overline{P}$

$L = rp\sin\theta$

$\overline{r}$: Position vector.

r sin θ : Perpendicular distance.

a. Relation between L & I , L= $I\omega$.
b. Linear momentum is measure of motion in linear motion.
c. Angular momentum is the measure of motion in rotation. Newton's second law for rotation $\overline{\tau} = \frac{d\overline{L}}{dt}$. Which gives rate of change of angular momentum is directly proportional to torque.
d. Relation between angular momentum and rotational K.E.

$$\therefore \quad KE = \frac{L^2}{2I}$$

12. For a rolling body it possess both rotational and transnational K.E.

$$E_K = E_t + E_r$$

$$E = \frac{1}{2}mr^2 + \frac{1}{2}I\omega^2$$

Where V: Velocity of centre of mass.

13. **Law of conservation of angular momentum:**
As long as external torque acting on a system its zero then the total angular momentum remains constant.

$$\overline{\tau} = \frac{d\overline{L}}{dt}.$$

If $\overline{\tau} = 0$

Where $\frac{d\overline{L}}{dt} = 0 \Rightarrow L$ is constant.

Where L = I ω, $\therefore$ I ω is constant

$I_1 \omega_1 = I_2 \omega_2$

14. **Theorems on moment of inertia**
a. Perpendicular axes theorem:
Statement: moment of inertia of a plane laminar about an axis perpendicular to its plane passing through a point is equal to the sum of moments of inertia of the lamina about any two mutually perpendicular axis in its plane and passing through same point.

$I_Z = I_x + I_y$.

b. Parallel axes theorem:
Moment of inertia of a rigid body about any axis is equal to the sum of its moment of inertia about a parallel axis passing through its centre of mass and the product of the mass of the body and square of the perpendicular distance between the two axes.

$I_Z = I_{Cm} + MR^2$

15. **Motion of a body in vertical circular plane:**
Important Formula:
a. Minimum velocity at lowest point to describe vertical circular motion

$$V_1 = \sqrt{5Rg}.$$

b. Critical velocity at highest point

$$V_2 = \sqrt{Rg}$$

c. Tension in the rope at lowest point

$$T_1 = \frac{mv_1^2}{R} + mg$$

d. Tension in the rope at the highest point

$$T_2 = \frac{mv_2^2}{R} - mg$$

e. At critical velocity tension at highest point in zero